

# Space-time symmetry is broken

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## Abstract

Space-time intervals corresponding to different events on the worldline of any ponderable object (for example a clock) are time-like. In consequence, in the analysis of any space-time experiment involving clocks only the region for  $c\Delta t \geq 0$  between the line  $\Delta x = 0$  and the light cone projection  $c\Delta t = \Delta x$  of the  $c\Delta t$  versus  $\Delta x$  Minkowski plot is physically relevant. This breaks the manifest space-time symmetry of the plot. A further consequence is the unphysical nature of the ‘relativity of simultaneity’ and ‘length contraction’ effects of conventional special relativity theory. The only modification of space-time transformation laws in passing from Galilean to special relativity is then the replacement of universal Newtonian time by a universal (position independent) time dilation effect for moving clocks.

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The concept of spontaneously broken symmetry is a ubiquitous one in modern physics. Originating in solid-state theory [1, 2], it is the basis of the Higgs mechanism of the standard model of particle physics [3]. As exemplified by the behaviour of a ferromagnet, spontaneous symmetry breaking occurs when the fundamental laws of some physical phenomenon respect a certain symmetry (rotational invariance in the case of a ferromagnet) which is broken in an actual physical realisation of the phenomenon. The fundamental laws are encapsulated in a Hamiltonian in non relativistic quantum mechanics, by a Lagrangian in relativistic quantum field theory, and by differential equations, such as Newton’s Second Law of mechanics, or Maxwell’s equations, in classical physics.

The fundamental laws of special relativity theory (SRT) are also encapsulated in differential equations, the Lorentz transformations (LT) for space and time intervals:

$$\Delta x' = \gamma(\Delta x - \beta \Delta x^0), \quad (1)$$

$$\Delta(x^0)' = \gamma(\Delta x^0 - \beta \Delta x) \quad (2)$$

where  $x^0 \equiv ct$ ,  $(x^0)' \equiv ct'$ ,  $\Delta x \equiv x_1 - x_2$  etc,  $\beta \equiv v/c$ ,  $\gamma \equiv 1/\sqrt{1 - \beta^2}$  and  $c$  is the speed of light in free space. The parallel  $x$  and  $x'$  coordinate axes are defined in the inertial frames S and S' respectively. The frame S' moves with speed  $v$  in the direction of the positive  $x$ -axis in S. Without any loss of generality, only points lying on the  $x, x'$  axes are considered in the following. The epochs  $t, t'$  are those recorded by similar clocks at rest in S, S' respectively.

The transformation equations (1) and (2) respect spatial and temporal translational invariance, that is they are unchanged by the replacements:

$$x \rightarrow x + X, \quad t \rightarrow t + T$$

where  $X$  and  $T$  are arbitrary constants. They also remain invariant under the operation of space-time exchange (STE):

$$x \leftrightarrow x^0, \quad x' \leftrightarrow (x^0)'$$

which exchanges equations (1) and (2). The STE invariance concept:

The equations describing the laws of physics are invariant with respect to the exchange of space and time coordinates, or, more generally to the exchange of the spatial and temporal components of four vectors.

was introduced in Ref. [4]. A corollary is the independence of physical predictions of any theory to the choice of metric (space-like or time-like) for four-vector products. As shown in Ref. [4] the postulate of STE invariance, together with the weak postulates of spatial homogeneity [5, 6, 7] or single-valuedness [8] is sufficient to derive the space-time LT (1) and (2). Another application of STE invariance is the derivation [4] of the non-homogeneous electrodynamical (Ampère's Law) and magnetodynamical (Faraday's Law of Induction) Maxwell equations from, respectively, the electrostatic and magnetostatic Gauss laws.

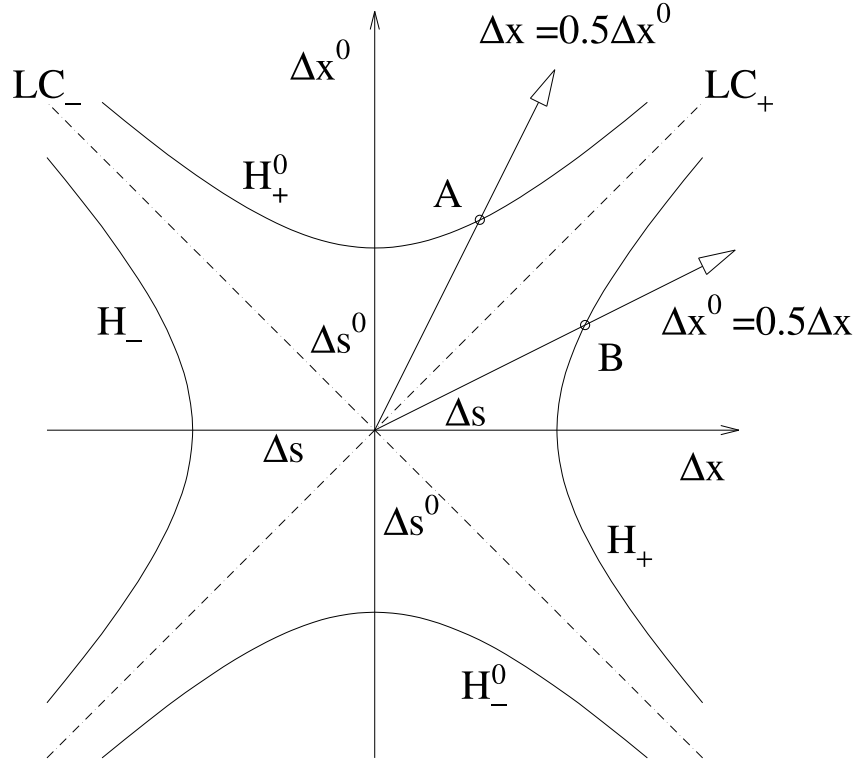


Figure 1: *Minkowski  $\Delta x^0$  versus  $\Delta x$  plot. The STE conjugate worldlines  $\Delta x = 0.5\Delta x^0, \Delta x^0 = 0.5\Delta x$  intersect the hyperbolae  $H_+^0, H_+$  corresponding, respectively, to time-like and space-like invariant interval relations, in the points A,B. See text for discussion.*

The transformation equations (1) and (2) may be combined to define invariant interval

equations as introduced by Minkowski [9]<sup>a</sup> and discussed at length by Langevin [11, 12]:

$$(\Delta x^0)^2 - (\Delta x)^2 = [\Delta(x^0)']^2 - (\Delta x')^2 \equiv (\Delta s^0)^2 \quad \text{Timelike interval} \quad \Delta x^0 > \Delta x, \quad (3)$$

$$(\Delta x)^2 - (\Delta x^0)^2 = (\Delta x')^2 - [\Delta(x^0)']^2 \equiv (\Delta s)^2 \quad \text{Spacelike interval} \quad \Delta x > \Delta x^0. \quad (4)$$

For a fixed value of  $\Delta s^0 = i\Delta s$  the intervals  $\Delta x^0(\beta)$ ,  $\Delta x(\beta)$  for different values of  $\beta$  lie along four distinct hyperbolae  $H_+$ ,  $H_-$ ,  $H_+^0$  and  $H_-^0$  in the  $\Delta x^0$  versus  $\Delta x$  Minkowski plot, as shown in Fig. 1. The equations of the hyperbolae are:

$$H_+ : \quad \Delta x^0 = \pm \sqrt{(\Delta x)^2 - (\Delta s)^2}, \quad \Delta x \geq \Delta s, \quad (5)$$

$$H_- : \quad \Delta x^0 = \pm \sqrt{(\Delta x)^2 - (\Delta s)^2}, \quad \Delta x \leq -\Delta s, \quad (6)$$

$$H_+^0 : \quad \Delta x = \pm \sqrt{(\Delta x^0)^2 - (\Delta s^0)^2}, \quad \Delta x^0 \geq \Delta s^0, \quad (7)$$

$$H_-^0 : \quad \Delta x = \pm \sqrt{(\Delta x^0)^2 - (\Delta s^0)^2}, \quad \Delta x^0 \leq -\Delta s^0. \quad (8)$$

Since the physical significance of Fig. 1 does not depend on the direction in which the  $\Delta x$  and  $\Delta x^0$  axes are drawn, the figure is invariant under the STE operation. In fact, the successive operations STE, anticlockwise rotation by  $90^\circ$  in the  $\Delta x\Delta x^0$  plane and rotation by  $180^\circ$  about the resulting  $\Delta x^0$  axis leave Fig. 1 unchanged. As will now be demonstrated, this manifest STE invariance is broken when the physical significance of various projection operators applied to the LT (1) and (2) is considered.

Setting  $\Delta x' = 0$  in Eq. (1) means consideration of events on the world line of a fixed point in the frame  $S'$ . The corresponding differential worldline equation in the frame  $S$  is, from Eq. (1),  $\Delta x = \beta \Delta x^0$ . For  $\beta = 0.5$  this straight line in Fig. 1 intersects the hyperbola  $H_+^0$  at the point A. Using the worldline equation in  $S$  to eliminate  $\Delta x$  in Eq. (2) yields the time dilation relation  $\Delta x^0 = \gamma \Delta(x^0)'$  which is the experimentally-confirmed [13, 14] prediction that clocks at rest in the frame  $S'$  are seen to run slow relative to clocks at rest in the frame  $S$ .

The STE conjugate projection  $\Delta(x^0)' = 0$ , i.e. simultaneous events in the frame  $S'$ , gives from Eq. (2) the relation  $\Delta x = \Delta x^0/\beta$  corresponding to a superluminal worldline in the frame  $S$  that intersects the hyperbola  $H_+$  in Fig. 1 at the point B for  $\beta = 0.5$ . Since for  $\Delta x > 0$  and  $\beta > 0$  then also  $\Delta x^0 > 0$ , there is here a ‘relativity of simultaneity’ effect because events simultaneous in  $S'$  ( $\Delta(x^0)' = 0$ ) are not so in the frame  $S$  ( $\Delta x^0 > 0$ ). Using the worldline equation to eliminate  $\Delta x^0$  in Eq. (1) gives  $\Delta x = \gamma \Delta x'$ . This is the ‘space dilation’ effect (the STE conjugate of time dilation) associated with the projection  $\Delta(x^0)' = 0$  as previously pointed out in Ref. [15]. However, *any object* at rest in the frame  $S'$  must have  $\Delta x' = 0$ . So it is impossible that the worldline of any physical clock at rest in  $S'$  can intersect the hyperbola  $H_+$ . The mathematical projection  $\Delta(x^0)' = 0$  with its associated relativity of simultaneity and ‘space dilation’ effects is therefore unphysical. The initial conditions of an experiment where a clock at rest in  $S'$  is compared with one at rest in  $S$ :  $\beta > 0$ ,  $\Delta x' = 0$  therefore restrict the physical region of the Minkowski plot in Fig. 1 to one eighth of its total area—that between the lightcone  $LC_+$  (the asymptote of the hyperbola  $H_+^0$ ) and the positive  $\Delta x^0$  axis. The symmetry of the plot is therefore clearly broken by the initial conditions of any experiment in which the time dilation effect is observed.

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<sup>a</sup>See Ref. [10] for a discussion of the consequences of a sign error in drawing the  $x'$  and  $t'$  axes on the original space-time plot of Ref. [9] as well as in a wide subsequent literature.

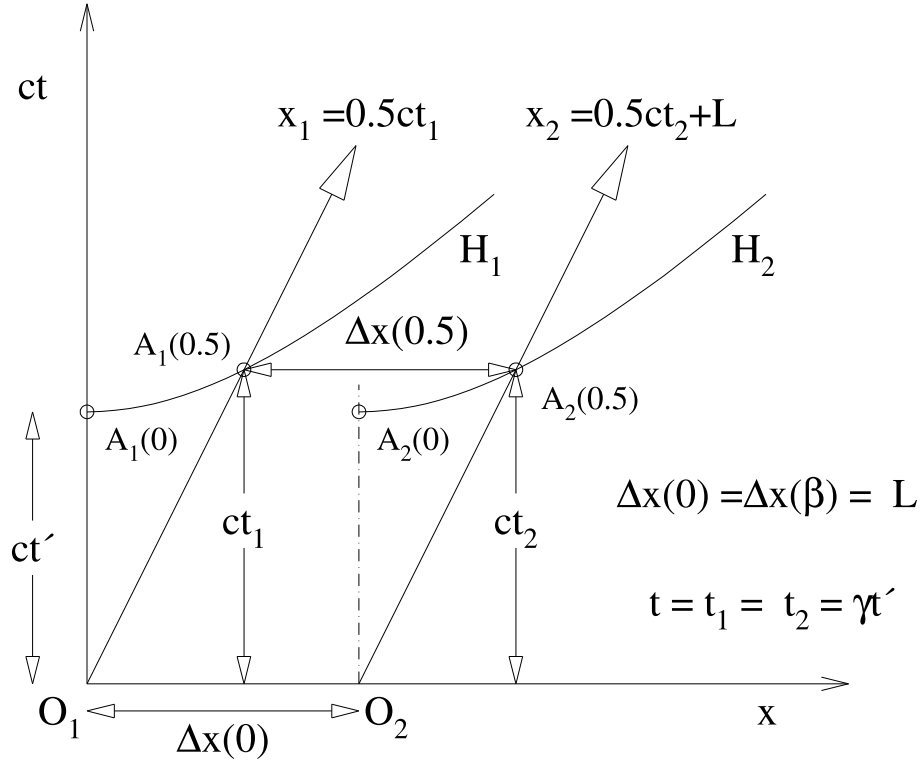


Figure 2: Minkowski  $ct$  versus  $x$  plot for clocks  $C_1$  and  $C_2$  at rest in the frame  $S'$  with respective worldlines:  $x_1 = \beta ct_1$  and  $x_2 = \beta ct_2 + L$  in the frame  $S$ . The hyperbolae  $H_1$  and  $H_2$  are the loci of points  $(x, ct)$  for a fixed value of  $t'$  and different positive values of  $\beta$ .  $O_1A_1(0.5)$ ,  $O_2A_2(0.5)$ : worldlines in  $S$  of  $C_1$  and  $C_2$  for  $\beta = 0.5$ .  $O_1A_1(0)$ ,  $O_2A_2(0)$ : similar wordlines for  $\beta = 0$ . The absence of any ‘relativity of simultaneity’ or ‘length contraction’ effects is evident from inspection of this figure.

Consider now two clocks  $C_1$  and  $C_2$  at rest in  $S'$  with worldlines in the frame  $S$ :  $x_1(\beta) = \beta ct_1$  and  $x_2(\beta) = \beta ct_2 + L$ . Integrating the differential time dilation relation  $dt = \gamma dt'$  gives time dilation relations  $t_1 = \gamma t'_1$ ,  $t_2 = \gamma t'_2$  for  $C_1$ ,  $C_2$ . Use of the identity  $\gamma^2(1 - \beta^2) \equiv 1$  then yields the relations:

$$c^2(t_1)^2 - x_1(\beta)^2 = c^2(t'_1)^2, \quad (9)$$

$$c^2(t_2)^2 - (x_2(\beta) - L)^2 = c^2(t'_2)^2. \quad (10)$$

The corresponding hyperbolae  $H_1$  and  $H_2$  on a  $ct$  versus  $x$  plot are shown in Fig. 2 for  $t'_1 = t'_2 = t'$ , together with the worldlines of  $C_1$  and  $C_2$  for  $\beta = 0.5$  and  $\beta = 0$ . It follows from the time dilation relations or inspection of Fig. 2 that  $t_1 = t_2$  when  $t'_1 = t'_2$ —there is no ‘relativity of simultaneity’ effect in observations of the clocks  $C_1$  and  $C_2$ . The worldline equations when  $t_1 = t_2$  show that, for all values of  $\beta$ :

$$\Delta x(\beta) \equiv x_2(\beta) - x_1(\beta) = L. \quad (11)$$

A special case of Eq. (12) is

$$x_2(0) - x_1(0) = x'_2 - x'_1 \equiv \Delta x' = x_2(\beta) - x_1(\beta) \equiv \Delta x(\beta) = L \quad (12)$$

so, as is also evident from inspection of Fig. 2, there is no ‘length contraction’ effect.

For further discussion of the invariance of measured length intervals—a property which is independent of the form of space-time transformation equations—see Ref. [16].

The interval LT (1) and (2) for the clock  $C_2$  are:

$$x'_2 - L = \gamma(x_2 - L - \beta ct_2) = 0, \quad (13)$$

$$ct'_2 = \gamma[ct_2 - \beta(x_2 - L)]. \quad (14)$$

The corresponding LT for clock  $C_1$  are given by setting  $L = 0$  in these equations.

It is now instructive to compare (13) and (14) with the conventional space-time LT [21]:

$$x' = \gamma(x - vt), \quad (15)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \quad (16)$$

which has hitherto been universally interpreted as the transformation giving the observed event in the frame  $S'$ :  $(x', t')$  corresponding to an event  $(x, t)$  observed in the frame  $S$ , for *arbitrary values* of  $x', t'$  or  $x, t$ . However, since the coordinate  $x'$  is, by definition, that of a fixed point in the frame  $S'$  it must be independent of time. In contrast, the right side of Eq. (15) is in general a function of the time  $t$  which, for any value of  $x$ , vanishes when  $t = x/v$ . It then necessarily follows that Eq. (15) *can hold only if both  $x' = 0$  and  $x = vt$* , in which case (15) and (16) become identical to (13) and (14) with  $L = 0$ , i.e. the correct interval LT for the clock  $C_1$  discussed above.

The spurious ‘length contraction’ and ‘relativity of simultaneity’ effects derived from (15) and (16), discussed in detail elsewhere [17, 18, 19], arise from the failure to respect the above-mentioned condition for the validity of Eq. (15). The LT (15) and (16) are instead assumed to hold for arbitrary values of  $x'$ , so they become, on considering two independent events:

$$x'_1 = \gamma(x_1 - vt_1), \quad x'_2 = \gamma(x_2 - vt_2), \quad (17)$$

$$t'_1 = \gamma\left(t_1 - \frac{vx_1}{c^2}\right), \quad t'_2 = \gamma\left(t_2 - \frac{vx_2}{c^2}\right). \quad (18)$$

On setting  $t_1 = t_2$  ( $\Delta t = 0$ , length measurement in the frame S) Eqs. (17) give:

$$x'_2 - x'_1 \equiv \Delta x' = \gamma(x_2 - x_1) \equiv \gamma\Delta x \quad (\text{length contraction})$$

while Eqs. (18) give:

$$t'_2 - t'_1 \equiv \Delta t' = -\frac{\gamma v(x_2 - x_1)}{c^2} = -\frac{\gamma\Delta x'}{c^2} \neq 0 \quad (\text{relativity of simultaneity}).$$

These unphysical predictions therefore arise from a failure to sufficiently consider the mathematical constraints arising from the operational meanings of the coordinate symbols in the LT.

The erroneous (when  $x' \neq 0$ ) LT equations (15) and (16) differ from the correct ones (13) and (14) by the omission of certain additive constants  $X$  and  $T$  on the right side of (15) and (16) respectively. As discussed in Ref. [20] the necessity to include such constants to correctly describe synchronised clocks at different spatial positions was clearly pointed out by Einstein in Ref. [21] but, to the present author's best knowledge, was never done, either by him or any subsequent worker, for the entire duration of the 20th Century!

The physical meaning of Eqs. (13) and (14) is the same as that of the more transparent equations:

$$x'_2 = L, \quad x_2 = vt_2 + L, \quad (19)$$

$$t_2 = \gamma t'_2. \quad (20)$$

The first and second equations in (19) are simply the worldline equations of  $C_2$  in the frames  $S'$ ,  $S$  respectively and are the same as in Galilean relativity. The only modification of space-time transformation equations in passing from Galilean to special relativity is the replacement of Newtonian universal time:  $T = t = t'$  by the position-independent time dilation relation (20).

Note that, as is also evident by inspection of Fig. 2, the worldlines of  $C_1$  and  $C_2$  in the frames  $S'$ ,  $S$  respectively, respect, at any instant:  $t = t_1 = t_2$  translational invariance:  $x'_2 = x'_1 + L$ ,  $x_2 = x_1 + L$ , as do the interval transformations for events on the worldlines of the clocks  $C_i$ ,  $i = 1, 2$ :

$$\Delta x'_i = 0, \quad \Delta x_i = \beta \Delta x_i^0, \quad (21)$$

$$\Delta x_i^0 = \gamma \Delta (x_i^0)'. \quad (22)$$

Comparing Eqs. (21) and (22) with the general, STE invariant, interval Lorentz transformations (1) and (2), the breakdown of STE invariance in space-time experiments involving such physical clocks is manifest.

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